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Technical Comments

Reply by the Authors to R. L. Glick and W. T. Brooks

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REFERENCE 1 compares two different motor temperature sensitivity relations. It purports to show that the most general relation[2,3]

$$\pi_K = \frac{\left(\dfrac{\partial \ln c}{\partial T}\right)_K + \ln p\left(\dfrac{\partial n}{\partial T}\right)_K + \pi_C}{1 - n} \tag{1}$$

is identical to the restricted relation[4] valid for only some propellants[2]

$$\pi_K = \frac{\sigma_p + \pi_C}{1 - n} \tag{2}$$

The first part of the discussion of Ref. 1 requires that $(\ )_p$ partial derivatives be equal to $(\ )_K$ partial derivatives. Using the definition of σ_p

$$\sigma_p = \left(\frac{\partial \ln c}{\partial T}\right)_p + \ln p\left(\frac{\partial n}{\partial T}\right)_p \tag{3}$$

in Eq. (2), then, Ref. 1 requires that

$$\left(\frac{\partial \ln c}{\partial T}\right)_K + \ln p\left(\frac{\partial n}{\partial T}\right)_K \overset{?}{=} \left(\frac{\partial \ln c}{\partial T}\right)_p + \ln p\left(\frac{\partial n}{\partial T}\right)_p = \sigma_p \tag{4}$$

The left-hand side (LHS) of Eq. (4) is **not** an equality since the definition of σ_p is that of Eq. (3), which does not involve $(\ )_K$ partial derivatives. For those benign propellants whose exponent is not a function of pressure, Eq. (4) may be written

$$\frac{\mathrm{d} \ln c}{\mathrm{d}T} + \ln p\frac{\mathrm{d}n}{\mathrm{d}T} = \sigma_p \tag{5}$$

since c and n are functions of temperature only. Thus, there are no partial derivatives, and either process may be assumed between the lower and upper temperatures. Nevertheless, when c and n become a function of pressure, it is absolutely necessary to distinguish between partial derivatives/processes. Equation (4) is not an equality, and Ref. 1 is wrong.

The second part of the discussion of Ref. 1 involves the use of an example of a specific propellant for which $n = n(p)$. When using a specific propellant, it becomes necessary to establish specific values for the initial and final conditions. The errors in Ref. 1 involve not using specific values. The specific conditions are, the pressure at point A, the initial condition on the curve[1] for T_1; the pressure at point B, the final condition on the curve[1] for T_2 at the end of a constant K process from point A; and the pressure at point C (NOT shown on the graph[1]), the final condition on the curve for T_2 at the end of a *constant pressure process* from point A. Reference 1 made an error in not locating point C on the graph, since σ_p used in Eq. (2) *can only involve* points A and C. As a result of the above errors, the several derived π_K equations[1] are wrong. The first error occurs in the values for n. Thus, for Eq. (2), n becomes[3]

$$n_C = 0.3 + 0.02 \ln p_C = 0.3 + 0.02 \ln p_A = n_A \tag{6}$$

since Ref. 1 indicates that

$$\left(\frac{\partial n}{\partial T}\right)_p = 0 \tag{7}$$

Conversely, the correct value of n for Eq. (1) is (since the final point[3] is B)

$$n_B = 0.3 + 0.02 \ln p_B \tag{8}$$

The n derivatives do not need specific values, as pressures and temperatures are not involved. However, the derivative for Eq. (1) should be written

$$\left(\frac{\partial \ln c}{\partial T}\right)_K = \frac{1}{T_2} - 0.02\pi_K \ln p_B \tag{9}$$

since the change in $\ln c$ is related to the final value of pressure, point B, on the T_2 rate curve (constant K process). Equation (9) predicts a decrease in the value of c compared to that at point C. The $\ln c$ partial derivative at constant pressure is merely equal to $1/T_2$, which predicts that the value of c will increase; obviously **not** correct for a rocket motor using this propellant[1]; one in which n increases with pressure and c decreases with pressure. As a result, the LHS of Eq. (4) becomes

$$(1/T_2) - 0.02\pi_K \ln p_B + \ln p_A[0.02\pi_K] \tag{10}$$

while the right-hand side (RHS) remains $1/T_2$. Clearly, then, Eq. (4) is not an equality, and the π_K equations are different. Using the previous relations yields for Eq. (1)

$$\pi_K = \frac{(1/T_2) + 0.02\pi_K(\ln p_A - \ln p_B) + \pi_C}{1 - 0.3 - 0.2 \ln p_B} \tag{11}$$

and for Eq. (2)

$$\pi_K = \frac{(1/T_2) + \pi_C}{1 - 0.3 - 0.2 \ln p_A} \tag{12}$$

Only two terms exist in the numerator of Eq. (12), neither of which is a function of the motor operation, clearly the result of Eq. (2) using σ_p, a propellant function only.

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Finally, it is not correct[3] to infer that the finite difference form of Eq. (2) is

$$\pi_K = \frac{\sigma_p + \pi_C}{1 - n} = \frac{1}{1 - n_B}\left[\frac{\ln(c_B/c_A)}{T_2 - T_1} + \ln p_A \frac{n_B - n_A}{T_2 - T_1} + \pi_C\right] \tag{13}$$

since Eq. (2) *can only involve* points A and C. The expression should be

$$\pi_K = \frac{\sigma_p + \pi_C}{1 - n} = \frac{1}{1 - n_C}\left[\frac{\ln(c_C/c_A)}{T_2 - T_1} + \pi_c\right] \tag{14}$$

since Eq. (7) eliminates the exponent term. On the other hand, the RHS of Eq. (13) is the correct expression[3] for Eq. (1).

Therefore, the two π_K equations are different. Equation (1) can predict rocket motor performance; Eq. (2) cannot. In summary, it may be concluded that the criticisms of Ref. 1 are incorrect.

References

[1]Glick, R. L., and Brooks, W. T., Comment on "Relationships for Temperature Sensitivity," *Journal of Propulsion and Power*, Vol. 10, No. 5, 1994, pp. 754, 755.

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[3]Hamke, R. E., and Osborn, J. R., "Relationships for Motor Temperature Sensitivity" (Errata), *Journal of Propulsion and Power*, Vol. 10, No. 1, 1994, p. 136.

[4]Glick, R. L., and Brooks, W. T., "Relations Among Temperature Sensitivity Parameters," *Journal of Propulsion and Power*, Vol. 1, No. 4, 1985, pp. 319, 320.

This Reply was not published in the same issue as the Comment because, through an editorial oversight, the authors of the Reply did not see the Comment until it appeared in print.

Comment on "Design of Axisymmetric Channels with Rotational Flow"

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KOUMANDAKIS et al.[1] have written an interesting paper on the "target pressure" problem. They use an inverse technique for the design of a duct with steady, inviscid flow. In particular, their paper deals with the subsonic, axisymmetric, rotational flow of a perfect gas, in which the Clebsch formulation is used.

The purpose of this Comment is to point out that the substitution principle[2] should provide a useful adjunct to their formulation. The two approaches appear to be complementary. For example, the basic assumptions are the same, both utilize the steady Euler equations for irrotational or rotational flows. Moreover, the substitution principle also uses a condition that is analogous to Eqs. (14) and (15) in Ref. 1. To the author's knowledge, the relationship between these two techniques has not been explored.

Since the substitution principle is not well known in fluid dynamics, we briefly outline this approach. The principle is a transformation of the dependent variables[2]

$$p = p_b, \qquad \rho = \lambda^{-1}\rho_b, \qquad h = \lambda h_b, \qquad w_i = \lambda^{1/2} w_{bi}$$

where p, ρ, h, and w_i are the pressure, density, enthalpy, and velocity components, respectively. A b subscript denotes an irrotational or rotational baseline solution of the steady Euler equations, which may be analytical, computational, or experimental. The unsubscripted variables represent a new, one-parameter family of solutions. The λ parameter must satisfy a streamline condition $D\lambda/Dt = 0$, where D/Dt is the substantial derivative. Typically, λ is the stagnation enthalpy or entropy, and to be useful, must vary in the directions transverse to the streamlines. If the flow contains shock waves, the additional restriction of a perfect gas is required. The new solutions satisfy the same wall-tangency condition as the original solution. Thus, the duct might first be designed for an irrotational flow.

While the principle holds for two-dimensional and axisymmetric flows, it also holds for three-dimensional flows that may contain supersonic regions with shock waves. Consequently, the principle may prove useful for obtaining three-dimensional solutions with or without shock waves, or for extending the number of such solutions.

References

[1]Koumandakis, M., Dedoussis, V., Chaviaropoulos, P., and Papailiou, K. D., "Design of Axisymmetric Channels with Rotational Flow," *Journal of Propulsion and Power*, Vol. 10, No. 5, 1994, pp. 729–735.

[2]Emanuel, G., *Analytical Fluid Dynamics*, CRC Press, Boca Raton, FL, 1994, pp. 161–179.

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